Management Science Lecture Notes | Chapter: 4

Modeling Examples

## A Product Mix Example

Quick-Screen is a clothing manufacturing company that specializes in producing commemorative shirts immediately following major sporting events such as the World Series, Super Bowl, and Final Four. The company has been contracted to produce a standard set of shirts for the winning team, either State University or Tech, following a college football bowl game on New Year's Day. The items produced include two sweatshirts, one with silk-screen printing on the front and one with print on both sides, and two T-shirts of the same configuration. The company has to complete all production within 72 hours after the game, at which time a trailer truck will pick up the shirts. The company will work around the clock. The truck has enough capacity to accommodate 1,200 standard-size boxes. A standard-size box holds 12 T-shirts, and a box of 12 sweatshirts is three times the size of a standard box. The company has budgeted $25,000 for the production run. It has 500 dozen blank sweatshirts and T-shirts each in stock, ready for production. This scenario is illustrated in Figure 4.1.

The resource requirements, unit costs, and profit per dozen for each type of shirt are shown in the following table:

| | Processing Time (hr.) per Dozen | Cost per Dozen | Profit per Dozen |
|---|---|---|---|
| ($x_1$) Sweatshirt—F | 0.10 | $36 | $ 90 |
| ($x_2$) Sweatshirt—B/F | 0.25 | 48 | 125 |
| ($x_3$) T-shirt—F | 0.08 | 25 | 45 |
| ($x_4$) T-shirt—B/F | 0.21 | 35 | 65 |

The company wants to know how many dozen (boxes) of each type of shirt to produce in order to maximize profit.

Answer; Maximize Profit

Subject to constraints; (i) Available time constraint: 72 hours
(ii) Available boxes constraint: 1200 boxes
(iii) Budget Constraint: 25000$
(iv) Stock for sweatshirts: 500
(v) Stock for T-shirts: 500

The LP model is:

Maximize $Z = 90x_1 + 125x_2 + 45x_3 + 65x_4$

subject to
(i) $0{,}10x_1 + 0{,}25x_2 + 0{,}08x_3 + 0{,}21x_4 \leq 72$
(ii) $3x_1 + 3x_2 + x_3 + x_4 \leq 1200$
(iii) $36x_1 + 48x_2 + 25x_3 + 35x_4 \leq 25000$
(iv) $x_1 + x_2 \leq 500$
(v) $x_3 + x_4 \leq 500$

$x_1, x_2, x_3, x_4 \geq 0$

The solution and sensitweety analysis can be followed ~~from~~ the tables below:

Original Problem

| | X1 | X2 | X3 | X4 | | RHS |
|---|---|---|---|---|---|---|
| Maximize | 90 | 125 | 45 | 65 | | |
| Processing Time | 0,1 | 0,25 | 0,08 | 0,21 | <= | 72 |
| Available Boxes | 3 | 3 | 1 | 1 | <= | 1200 |
| Budget | 36 | 48 | 25 | 35 | <= | 25000 |
| Sweetshirt Stock | 1 | 1 | 0 | 0 | <= | 500 |
| T-shirt Stock | 0 | 0 | 1 | 1 | <= | 500 |
| Solution-> | 175,5556 | 57,7778 | 500 | 0 | Optimal Z-> | 45522,22 |

Ranging

| Variable | Value | Reduced Cost | Original Val | Lower Bound | Upper Bound |
|---|---|---|---|---|---|
| X1 | 175,5556 | 0 | 90 | 50 | 101,9231 |
| X2 | 57,7778 | 0 | 125 | 113,0769 | 138,2143 |
| X3 | 500 | 0 | 45 | 40,8889 | Infinity |
| X4 | 0 | 10,3333 | 65 | -Infinity | 75,3333 |

| Constraint | Dual Value | Slack/Surplus | Original Val | Lower Bound | Upper Bound |
|---|---|---|---|---|---|
| Processing Time | 233,3333 | 0 | 72 | 63,3333 | 98,3333 |
| Available Boxes | 22,2222 | 0 | 1200 | 884 | 1460 |
| Budget | 0 | 3406,666 | 25000 | 21593,33 | Infinity |
| Sweetshirt Stock | 0 | 266,6667 | 500 | 233,3333 | Infinity |
| T-shirt Stock | 4,1111 | 0 | 500 | 0 | 685,7144 |


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The model Solution is;

- Maximum profit is $z = 45522{,}22\,\$$ → 45375 \$ when we produce 175,56 (actually 175) Sweatshirt-F, 57,78 (actually 57) Sweatshirt-B/F, 500 T-shirt-F and NO T-shirt B/F.

- All of the Processing Time, Available Boxes and T-Shirt Stock Sources are used up. 266 Sweatshirt stock and 3406\$ budget is NOT used.

- Without changing the solution, we can decrease the profit of Sweatshirt-F to 50\$ in bad economy and increase it to 104,92\$ in good economy. Likewise, Sweatshirt-B/F profit can be decreased to 113,07\$ and increased to 138,21 \$. T-shirt-F price can be decreased to 40,88\$ and there's NO upper bound for increase. Since we do NOT produce T-shirt-B/F, the solution would NOT change for profit of T-Shirt B/F up to 75,33 \$.

- An increase in Budget or Sweatshirt Stock available will NOT increase the maximum profit, since their Dual Values are 0. (Note that they're NOT used up!) Let's say, Processing time available has increased to 90 hours. Our new profit will be;

$$z = 45522{,}22 + (90-72) \cdot 233{,}33 = 49722{,}16$$



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## A Diet Example

Breathtakers, a health and fitness center, operates a morning fitness program for senior citizens. The program includes aerobic exercise, either swimming or step exercise, followed by a healthy breakfast in the dining room. Breathtakers' dietitian wants to develop a breakfast that will be high in calories, calcium, protein, and fiber, which are especially important to senior citizens, but low in fat and cholesterol. She also wants to minimize cost. She has selected the following possible food items, whose individual nutrient contributions and cost from which to develop a standard breakfast menu are shown in the following table:

| Breakfast Food | Calories | Fat (g) | Cholesterol (mg) | Iron (mg) | Calcium (mg) | Protein (g) | Fiber (g) | Cost |
|---|---|---|---|---|---|---|---|---|
| 1. Bran cereal (cup) | 90 | 0 | 0 | 6 | 20 | 3 | 5 | $0.18 |
| 2. Dry cereal (cup) | 110 | 2 | 0 | 4 | 48 | 4 | 2 | 0.22 |
| 3. Oatmeal (cup) | 100 | 2 | 0 | 2 | 12 | 5 | 3 | 0.10 |
| 4. Oat bran (cup) | 90 | 2 | 0 | 3 | 8 | 6 | 4 | 0.12 |
| 5. Egg | 75 | 5 | 270 | 1 | 30 | 7 | 0 | 0.10 |
| 6. Bacon (slice) | 35 | 3 | 8 | 0 | 0 | 2 | 0 | 0.09 |
| 7. Orange | 65 | 0 | 0 | 1 | 52 | 1 | 1 | 0.40 |
| 8. Milk—2% (cup) | 100 | 4 | 12 | 0 | 250 | 9 | 0 | 0.16 |
| 9. Orange juice (cup) | 120 | 0 | 0 | 0 | 3 | 1 | 0 | 0.50 |
| 10. Wheat toast (slice) | 65 | 1 | 0 | 1 | 26 | 3 | 3 | 0.07 |

The dietitian wants the breakfast to include at least 420 calories, 5 milligrams of iron, 400 milligrams of calcium, 20 grams of protein, and 12 grams of fiber. Furthermore, she wants to limit fat to no more than 20 grams and cholesterol to 30 milligrams.

Answer; Minimize Cost

Subject to constraints:

(i) At least 420 calories

(ii) At least 5mg of iron

(iii) At least 400mg. of Calcium

(iv) At least 20gr. of Protein

(v) At least 12gr. of fiber

(vi) At most 20gr. of fat

(vii) At most 30mg of cholesterol

The LP Model is;

Maximize $Z = 0{,}18x_1 + 0{,}22x_2 + 0{,}10x_3 + 0{,}12x_4 + 0{,}10x_5 + 0{,}09x_6 + 0{,}40x_7 + 0{,}16x_8 + 0{,}50x_9 + 0{,}07x_{10}$

Subject to:

$90x_1 + 110x_2 + 100x_3 + 90x_4 + 75x_5 + 35x_6 + 65x_7 + 100x_8 + 120x_9 + 65x_{10} \geq 420$

$6x_1 + 4x_2 + 2x_3 + 3x_4 + x_5 + \quad x_7 + \quad x_{10} \geq 5$

$$20x_1 + 48x_2 + 12x_3 + 8x_4 + 30x_5 + 52x_7 + 250x_8 + 3x_9 + 26x_{10} \geq 400$$

$$3x_1 + 4x_2 + 5x_3 + 6x_4 + 7x_5 + 2x_6 + x_7 + 9x_8 + x_9 + 3x_{10} \geq 20$$

$$5x_1 + 2x_2 + 3x_3 + 4x_4 + x_7 + 3x_{10} \geq 12$$

$$2x_2 + 2x_3 + 2x_4 + 5x_5 + 3x_6 + 4x_8 + x_{10} \leq 20$$

$$270x_5 + 8x_6 + 12x_8 \leq 30$$

$$x_1, x_2, \ldots, x_{10} \geq 0$$

## A Marketing Example

The Biggs Department Store chain has hired an advertising firm to determine the types and amount of advertising it should invest in for its stores. The three types of advertising available are television and radio commercials and newspaper ads. The retail chain desires to know the number of each type of advertisement it should purchase in order to maximize exposure. It is estimated that each ad or commercial will reach the following potential audience and cost the following amount:

| | | Exposure (people/ad or commercial) | Cost |
|---|---|---|---|
| $x_1$: | Television commercial | 20,000 | \$15,000 |
| $x_2$: | Radio commercial | 12,000 | 6,000 |
| $x_3$: | Newspaper ad | 9,000 | 4,000 |

The company must consider the following resource constraints:

1. The budget limit for advertising is \$100,000.
2. The television station has time available for 4 commercials.
3. The radio station has time available for 10 commercials.
4. The newspaper has space available for 7 ads.
5. The advertising agency has time and staff available for producing no more than a total of 15 commercials and/or ads.

Answer: The LP model is;

Maximize $Z = 20000x_1 + 12000x_2 + 9000x_3$

Subject to:

$$15000x_1 + 6000x_2 + 4000x_3 \leq 100000$$

$$x_1 \leq 4$$

$$x_2 \leq 10$$

$$x_3 \leq 7$$

$$x_1 + x_2 + x_3 \leq 15$$

The solution and sensitiveety analysis can be followed from the tables below:

Orginal Problem

| | X1 | X2 | X3 | | RHS |
|---|---|---|---|---|---|
| **Maximize** | 20000 | 12000 | 9000 | | |
| **Budget Limit** | 15000 | 6000 | 4000 | <= | 100000 |
| **Television ADs** | 1 | 0 | 0 | <= | 4 |
| **Radio ADs** | 0 | 1 | 0 | <= | 10 |
| **Newspaper ADs** | 0 | 0 | 1 | <= | 7 |
| **Total ADs** | 1 | 1 | 1 | <= | 15 |
| **Solution->** | 1,8182 | 10 | 3,1818 | Optimal Z-> | 185000 |

Ranging

| Variable | Value | Reduced Cost | Original Val | Lower Bound | Upper Bound |
|---|---|---|---|---|---|
| **X1** | **1,8182** | 0 | 20000 | 9000 | 25500 |
| **X2** | **10** | 0 | 12000 | 11000 | Infinity |
| **X3** | **3,1818** | 0 | 9000 | 5333,333 | 10222,22 |
| **Constraint** | **Dual Value** | **Slack/Surplus** | **Original Val** | **Lower Bound** | **Upper Bound** |
| **Budget Limit** | **1** | 0 | 100000 | 80000 | 124000 |
| **Television ADs** | **0** | 2,1818 | 4 | 1,8182 | Infinity |
| **Radio ADs** | **1000** | 0 | 10 | 5,3333 | 13,8889 |
| **Newspaper ADs** | **0** | 3,8182 | 7 | 3,1818 | Infinity |
| **Total ADs** | **5000** | 0 | 15 | 12,6667 | 17,8 |

- We reach 185000 (actually 167000) people when we make 1,818 (1) television commercials, 10 radio commerkials and 3,182 (3) newspaper advertisements. We used all budget limit, max. of Radio ads and Max. of Total ads wheeas 2,18 (3) Television ads and 3,818 (4) Newspaper ads remains unused.
- If we were going to choose within one extra ads among three, our choice would be Radio ADs because it's Dual Value (in constraint table) is highest.



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• If Number of people we can reach by Television changes within the ~~[illegible]~~ interval (9000 ; 25500), we would NOT change our solution $x_1 = 1,818$; $x_2 = 10$; $x_3 = 3,182$.
Then, for example, let the Number of people we can reach by Televistion decreased to 16000. What is the new optimum solution?

$$New\ z = 16000 \cdot 1,8182 + 12000 \cdot 10 + 9000\, 3,1818 = 17\cancel{7}727 \text{ people}$$

• Let, total AD's available increased to 17. What is its effect to optimal solution?

We have,

$$New\ z = \underbrace{185000}_{\text{old } z} + (\underbrace{17}_{\text{New Total}} - \underbrace{15}_{\text{old Total}}) \cdot \underbrace{5000}_{\text{Total AD's Dual Value}} = 195000$$

Change in available Total AD's

Change in optimal z.

Question: Since we must have an integer solution, we used $x_1 = 1$, $x_2 = 10$ and $x_3 = 3$ ADs, ~~[illegible]~~ and reached 167000 people. A new manager offered the solution $x_1 = 2$, $x_2 = 9$ and $x_3 = 4$ and claimed that this new solution is optimal. Check this claim.

Answer (i) Check the constraints:

$$15000 \cdot 2 + 6000 \cdot 9 + 4000 \cdot 4 = 100000 \le 100000 \checkmark$$

$$x_1 = 2 \le 4 \checkmark \quad x_2 = 9 \le 10 \checkmark \quad x_3 = 4 \le 7 \checkmark$$

$$x_1 + x_2 + x_3 = 1 + 10 + 3 = 14 \le 15 \checkmark$$

(ii) Check New optimal solution.

$$New\ z = 20000 \cdot 2 + 12000 \cdot 9 + 9000 \cdot 4 = 184000 > 167000$$

The claim is TRUE!



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## A Transportation Example

The Zephyr Television Company ships televisions from three warehouses to three retail stores on a monthly basis. Each warehouse has a fixed supply per month, and each store has a fixed demand per month. The manufacturer wants to know the number of television sets to ship from each warehouse to each store in order to minimize the total cost of transportation.

Each warehouse has the following supply of televisions available for shipment each month:

| *Warehouse* | *Supply (sets)* |
|---|---|
| 1. Cincinnati | 300 |
| 2. Atlanta | 200 |
| 3. Pittsburgh | 200 |
| | 700 |

Each retail store has the following monthly demand for television sets:

| *Store* | *Demand (sets)* |
|---|---|
| A. New York | 150 |
| B. Dallas | 250 |
| C. Detroit | 200 |
| | 600 |

Costs of transporting television sets from the warehouses to the retail stores vary as a result of differences in modes of transportation and distances. The shipping cost per television set for each route is as follows:

| From Warehouse | To Store A | B | C | Supply: |
|---|---|---|---|---|
| 1 | \$16 | \$18 | \$11 | 300 |
| 2 | 14 | 12 | 13 | 200 |
| 3 | 13 | 15 | 17 | 200 |
| Demand: | 150 | 250 | 200 | |

$\sum s_i = 700$

$\sum d_i = 600$

$\neq$ "UNBALANCED" problem.

Answer; The decision Variables ARE:

$x_{ij}$: Number of television sets shipped from warehouse $i$ to store $j$; $i = 1, 2, 3$; $j = A, B, C$

The LP Model is;

Minimize $Z = 16x_{1A} + 18x_{1B} + 11x_{1C} + 14x_{2A} + 12x_{2B} + 13x_{2C} + 13x_{3A} + 15x_{3B} + 17x_{3C}$

subject to:

SUPPLY
(i) $x_{1A} + x_{1B} + x_{1C} \leq 300$
(ii) $x_{2A} + x_{2B} + x_{2C} \leq 200$
(iii) $x_{3A} + x_{3B} + x_{3C} \leq 200$

DEMAND
(iv) $x_{1A} + x_{2A} + x_{3A} = 150$
(v) $x_{1B} + x_{2B} + x_{3B} = 250$
(vi) $x_{1C} + x_{2C} + x_{3C} = 200$

$x_{ij} \geq 0$
$i = 1, 2, 3; j = A, B, C$

The solution of the Transportation problem is in the second table below:

| Data | STORE | | | |
|---|---|---|---|---|
| COSTS | New York | Dallas | Detroid | Supply |
| Cincinnati | 16 | 18 | 11 | 300 |
| Atlanta | 14 | 12 | 13 | 200 |
| Pittsburg | 13 | 15 | 17 | 200 |
| Demand | 150 | 250 | 200 | 600 \ 700 |

WAREHOUSE

| Shipments | STORE | | | |
|---|---|---|---|---|
| Shipments | New York | Dallas | Detroid | Row Total |
| Cincinnati | 0 | 0 | 200 | 200 |
| Atlanta | 0 | 200 | 0 | 200 |
| Pittsburg | 150 | 50 | 0 | 200 |
| Column Total | 150 | 250 | 200 | 600 \ 600 |

WAREHOUSE

**Total Cost** **7300**

We have;

$x_{1C} = 200$ TV → from 1 to C

$x_{2B} = 200$ TV → from 2 to B

$x_{3A} = 150$ TV → from 3 to A

$x_{3B} = 50$ TV → from 3 to B

$x_{ij} = 0$ for others

Total (Minimum) cost is $Z = 7300$.

## The Assignment Model

*An assignment model is for a special form of transportation problem in which all supply and demand values equal one.*

The **assignment model** is a special form of a linear programming model that is similar to the transportation model. There are differences, however. In the assignment model, the supply at each source and the demand at each destination are each limited to one unit.

The following example will demonstrate the assignment model. The Atlantic Coast Conference (ACC) has four basketball games on a particular night. The conference office wants to assign four teams of officials to the four games in a way that will minimize the total distance traveled by the officials. The supply is always one team of officials, and the demand is for only one team of officials at each game. The distances in miles for each team of officials to each game location are shown in the following table:

The travel distances to each game for each team of officials

| | Game Sites | | | |
|---|---|---|---|---|
| Officials | (1) Raleigh | (2) Atlanta | (3) Durham | (4) Clemson |
| A | 210 | 90 | 180 | 160 |
| B | 100 | 70 | 130 | 200 |
| C | 175 | 105 | 140 | 170 |
| D | 80 | 65 | 105 | 120 |

Formulate an LP which assigns officials to Game Sites at minimum distance.

Answer: The decision variables are;

$$x_{ij} = \begin{cases} 1 & \text{if official } i \text{ is assigned to game site } j \\ 0 & \text{otherwise} \end{cases}$$

where $i = A, B, C, D$; $j = 1, 2, 3, 4$

Minimize
$$z = 210x_{A1} + 90x_{A2} + 180x_{A3} + 160x_{A4} + 100x_{B1} + 70x_{B2} + 130x_{B3} + 200x_{B4} + 175x_{C1} + 105x_{C2} + 140x_{C3} + 170x_{C4} + 80x_{D1} + 65x_{D2} + 105x_{D3} + 120x_{D4}$$

subject to;

Officials

(i) $x_{A1} + x_{A2} + x_{A3} + x_{A4} = 1$

(ii) $x_{B1} + x_{B2} + x_{B3} + x_{B4} = 1$

(iii) $x_{C1} + x_{C2} + x_{C3} + x_{C4} = 1$

(iv) $x_{D1} + x_{D2} + x_{D3} + x_{D4} = 1$

Game sites

(v) $x_{A1} + x_{B1} + x_{C1} + x_{D1} = 1$

(vi) $x_{A2} + x_{B2} + x_{C2} + x_{D2} = 1$

(vii) $x_{A3} + x_{B3} + x_{C3} + x_{D3} = 1$

(viii) $x_{A4} + x_{B4} + x_{C4} + x_{D4} = 1$

$x_{ij} \geq 0$ for $i = A, B, C, D$; $j = 1, 2, 3, 4$

**Assignments**

| Shipments | (1) RALEIGH | (2) ATLANTA | (3) DURHAM | (4) CLEMSON | Row Total |
|---|---|---|---|---|---|
| OFFICIAL A | | 1 | | | 1 |
| OFFICIAL B | 1 | | | | 1 |
| OFFICIAL C | | | 1 | | 1 |
| OFFICIAL D | | | | 1 | 1 |
| Column Total | 1 | 1 | 1 | 1 | 4 |

**Total Cost** **450**

The solution is;

Minimum (TOTAL) Distance is $z = 450$ when

$x_{A2} = x_{B1} = x_{C3} = x_{D4} = 1$ and other $x_{ij} = 0$. Namely,

Assign official A to Atlanta; official B to Raleigh; official C to Durham and official D to Clemson to reach minimum available distance 450.



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7. Betty Malloy, owner of the Eagle Tavern in Pittsburgh, is preparing for Super Bowl Sunday, and she must determine how much beer to stock. Betty stocks three brands of beer—Yodel, Shotz, and Rainwater. The cost per gallon (to the tavern owner) of each brand is as follows:

| | Brand | Cost/Gallon |
|---|---|---|
| ($x_1$) | Yodel | \$1.50 |
| ($x_2$) | Shotz | 0.90 |
| ($x_3$) | Rainwater | 0.50 |

The tavern has a budget of \$2,000 for beer for Super Bowl Sunday. Betty sells Yodel at a rate of \$3.00 per gallon, Shotz at \$2.50 per gallon, and Rainwater at \$1.75 per gallon. Based on past football games, Betty has determined the maximum customer demand to be 400 gallons of Yodel, 500 gallons of Shotz, and 300 gallons of Rainwater. The tavern has the capacity to stock 1,000 gallons of beer; Betty wants to stock up completely. Betty wants to determine the number of gallons of each brand of beer to order so as to maximize profit.

Formulate a linear programming model for this problem.

7) Maximize $z = \overset{3-1,5}{1,5x_1} + \overset{2,5-0,9}{1,6x_2} + \overset{1,7-0,5}{1,2x_3}$

Subject to $1,5x_1 + 0,9x_2 + 0,5x_3 \leq 2000$

$x_1 \leq 400$

$x_2 \leq 500$

$x_3 \leq 300$

$x_1 + x_2 + x_3 \geq 1000$

$x_1, x_2, x_3 \geq 0$

8. The Kalo Fertilizer Company produces two brands of lawn fertilizer—Super Two and Green Grow—at plants in Fresno, California, and Dearborn, Michigan. The plant at Fresno has resources available to produce 5,000 pounds of fertilizer daily; the plant at Dearborn has enough resources to produce 6,000 pounds daily. The cost per pound of producing each brand at each plant is as follows:

| | Plant | |
|---|---|---|
| Product | Fresno | Dearborn |
| Super Two | \$2 | \$4 |
| Green Grow | 2 | 3 |

The company has a daily budget of \$45,000 for both plants combined. Based on past sales, the company knows the maximum demand (converted to a daily basis) is 6,000 pounds for Super Two and 7,000 pounds for Green Grow. The selling price is \$9 per pound for Super Two and \$7 per pound for Green Grow. The company wants to know the number of pounds of each brand of fertilizer to produce at each plant in order to maximize profit.

Formulate a linear programming model for this problem.

8) The decision variables are;

| | Fresno | Dearborn | DEMAND |
|---|---|---|---|
| [9] Super Two | (2) $x_1$ | (4) $x_2$ | 6000 |
| [7] Green Grow | (2) $x_3$ | (3) $x_4$ | 7000 |
| AVAILABLE | 5000 | 6000 | |

(c): Cost / pound
[s]: Sale price / pound

$$\text{Maximize} \quad Z = \overset{9-2}{7}x_1 + \overset{9-4}{5}x_2 + \overset{7-2}{5}x_3 + \overset{7-3}{4}x_4$$

$$\text{Subject to} \quad 2x_1 + 4x_2 + 2x_3 + 3x_4 \leq 45000$$

$$x_1 + x_3 \leq 5000$$

$$x_2 + x_4 \leq 6000$$

$$x_1 + x_2 \leq 6000$$

$$x_3 + x_4 \leq 7000$$

$$x_1, x_2, x_3, x_4 \geq 0$$

13. Anna Broderick is the dietitian for the State University football team, and she is attempting to determine a nutritious lunch menu for the team. She has set the following nutritional guidelines for each lunch serving:

- Between 1,500 and 2,000 calories
- At least 5 mg of iron
- At least 20 but no more than 60 g of fat
- At least 30 g of protein
- At least 40 g of carbohydrates
- No more than 30 mg of cholesterol

She selects the menu from seven basic food items, as follows, with the nutritional contribution per pound and the cost as given:

| | Calories (per lb.) | Iron (mg/lb.) | Protein (g/lb.) | Carbohydrates (g/lb.) | Fat (g/lb.) | Cholesterol (mg/lb.) | $/lb. |
|---|---|---|---|---|---|---|---|
| $x_1$ Chicken | 520 | 4.4 | 17 | 0 | 30 | 180 | 0.80 |
| $x_2$ Fish | 500 | 3.3 | 85 | 0 | 5 | 90 | 3.70 |
| $x_3$ Ground beef | 860 | 0.3 | 82 | 0 | 75 | 350 | 2.30 |
| $x_4$ Dried beans | 600 | 3.4 | 10 | 30 | 3 | 0 | 0.90 |
| $x_5$ Lettuce | 50 | 0.5 | 6 | 0 | 0 | 0 | 0.75 |
| $x_6$ Potatoes | 460 | 2.2 | 10 | 70 | 0 | 0 | 0.40 |
| $x_7$ Milk (2%) | 240 | 0.2 | 16 | 22 | 10 | 20 | 0.83 |

The dietitian wants to select a menu to meet the nutritional guidelines while minimizing the total cost per serving.

Formulate a linear programming model for this problem.

13) Minimize $Z = 0{,}80x_1 + 3{,}70x_2 + 2{,}30x_3 + 0{,}90x_4$
$+ 0{,}75x_5 + 0{,}40x_6 + 0{,}83x_7$

Subject to:

$520x_1 + 500x_2 + 860x_3 + 600x_4 + 50x_5 + 460x_6 + 240x_7 \geq 1500$
$520x_1 + 500x_2 + 860x_3 + 600x_4 + 50x_5 + 460x_6 + 240x_7 \leq 2000$
$4{,}4x_1 + 3{,}3x_2 + 0{,}3x_3 + 3{,}4x_4 + 0{,}5x_5 + 2{,}2x_6 + 0{,}2x_7 \geq 5$
$30x_1 + 5x_2 + 75x_3 + 3x_4 + 10x_7 \geq 20$
$30x_1 + 5x_2 + 75x_3 + 3x_4 + 10x_7 \leq 60$
$17x_1 + 85x_2 + 82x_3 + 10x_4 + 6x_5 + 10x_6 + 16x_7 \geq 30$
$30x_4 + 70x_6 + 22x_7 \geq 40$
$+ 20x_7 \leq 30$
$180x_1 + 90x_2 + 350x_3$

14. The Cabin Creek Coal (CCC) Company operates three mines in Kentucky and West Virginia, and it supplies coal to four utility power plants along the East Coast. The cost of shipping coal from each mine to each plant, the capacity at each of the three mines, and the demand at each plant are shown in the following table:

| | Plant | | | | |
|---|---|---|---|---|---|
| Mine | 1 | 2 | 3 | 4 | Mine Capacity (tons) |
| 1 | \$7 | \$9 | \$10 | \$12 | 220 |
| 2 | 9 | 7 | 8 | 12 | 170 |
| 3 | 11 | 14 | 5 | 7 | 280 |
| Demand (tons) | 110 | 160 | 90 | 180 | |

The cost of mining and processing coal is \$62 per ton at mine 1, \$67 per ton at mine 2, and \$75 per ton at mine 3. The percentage of ash and sulfur content per ton of coal at each mine is as follows:

| Mine | % Ash | % Sulfur |
|---|---|---|
| 1 | 9 | 6 |
| 2 | 5 | 4 |
| 3 | 4 | 3 |

Each plant has different cleaning equipment. Plant 1 requires that the coal it receives have no more than 6% ash and 5% sulfur; plant 2 coal can have no more than 5% ash and sulfur combined; plant 3 can have no more than 5% ash and 7% sulfur; and plant 4 can have no more than 6% ash and sulfur combined. CCC wants to determine the amount of coal to produce at each mine and ship to its customers that will minimize its total cost.

Formulate a linear programming model for this problem.

14) Decision Variables are;

$x_{ij}$ : Coal carried from mine $i$ to plant $j$; $i = 1,2,3$, $j = 1,2,3,4$

Minimize $z = 62(x_{11}+x_{12}+x_{13}+x_{14}) + 67(x_{21}+x_{22}+x_{23}+x_{24})$

$+ 75(x_{31}+x_{32}+x_{33}+x_{34}) + 7x_{11} + 9x_{12} + 10x_{13} + 12x_{14}$

$+ 9x_{21} + 7x_{22} + 8x_{23} + 14x_{24} + 11x_{31} + 14x_{32} + 5x_{33} + 7x_{34}$

Subject to

$$\left.\begin{aligned} x_{11}+x_{12}+x_{13}+x_{14} &\le 220 \\ x_{21}+x_{22}+x_{23}+x_{24} &\le 170 \\ x_{31}+x_{32}+x_{33}+x_{34} &\le 280 \end{aligned}\right\}$$

Supply Constraints

$\sum s_i = 220+170+280 = 670$

$$\left.\begin{aligned} x_{11}+x_{21}+x_{31} &= 110 \\ x_{12}+x_{22}+x_{32} &= 160 \\ x_{13}+x_{23}+x_{33} &= 90 \\ x_{14}+x_{24}+x_{34} &= 180 \end{aligned}\right\}$$

Demand Constraints

$\sum d_j = 110+160+90+180 = 540$

| Plant | Ash | Sulfur |
|---|---|---|
| 1 | 6 | 5 |
| 2 | 5 | 5 |
| 3 | 5 | 7 |
| 4 | 6 | 6 |

$$\frac{9x_{11}+5x_{21}+4x_{31}}{x_{11}+x_{21}+x_{31}} \le 6 \qquad \frac{6x_{11}+4x_{21}+3x_{31}}{x_{11}+x_{21}+x_{31}} \le 5$$

$$\frac{9x_{12}+5x_{22}+4x_{32}}{x_{12}+x_{22}+x_{32}} \le 5 \qquad \frac{6x_{12}+4x_{22}+3x_{32}}{x_{12}+x_{22}+x_{32}} \le 5$$

$$\frac{9x_{13}+5x_{23}+4x_{33}}{x_{13}+x_{23}+x_{33}} \le 5 \qquad \frac{6x_{13}+4x_{23}+3x_{33}}{x_{13}+x_{23}+x_{33}} \le 7$$

$$\frac{9x_{14}+5x_{24}+4x_{34}}{x_{14}+x_{24}+x_{34}} \le 6 \qquad \frac{6x_{14}+4x_{24}+3x_{34}}{x_{14}+x_{24}+x_{34}} \le 6$$

$x_{ij} \ge 0$



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19. A publishing house publishes three weekly magazines—*Daily Life*, *Agriculture Today*, and *Surf's Up*. Publication of one issue of each of the magazines requires the following amounts of production time and paper:

| | Production (hr.) | Paper (lb.) |
|---|---|---|
| $x_1$ *Daily Life* | 0.01 | 0.2 |
| $x_2$ *Agriculture Today* | 0.03 | 0.5 |
| $x_3$ *Surf's Up* | 0.02 | 0.3 |

Each week the publisher has available 120 hours of production time and 3,000 pounds of paper. Total circulation for all three magazines must exceed 5,000 issues per week if the company is to keep its advertisers. The selling price per issue is \$2.25 for *Daily Life*, \$4.00 for *Agriculture Today*, and \$1.50 for *Surf's Up*. Based on past sales, the publisher knows that the maximum weekly demand for *Daily Life* is 3,000 issues; for *Agriculture Today*, 2,000 issues; and for *Surf's Up*, 6,000 issues. The production manager wants to know the number of issues of each magazine to produce weekly in order to maximize total sales revenue.

Formulate a linear programming model for this problem.

19)

| | Production | Paper | PROFIT |
|---|---|---|---|
| ($x_1$) Daily life | 0,01 | 0,2 | 2,25 |
| ($x_2$) Agriculture Today | 0,03 | 0,5 | 4 |
| ($x_3$) Surf's Up | 0,02 | 0,3 | 1,5 |
| AVAILABLE | 120 | 3000 | |

$$\text{maximize} \quad z = 2{,}25x_1 + 4x_2 + 1{,}5x_3$$

$$\text{subject to} \quad 0{,}01x_1 + 0{,}03x_2 + 0{,}02x_3 \le 120$$

$$0{,}2x_1 + 0{,}5x_2 + 0{,}3x_3 \le 3000$$

$$x_1 + x_2 + x_3 \ge 5000$$

$$x_1 \le 3000$$

$$x_2 \le 2000$$

$$x_3 \le 6000$$

$$x_1, x_2, x_3 \ge 0$$



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20. The manager of a department store in Seattle is attempting to decide on the types and amounts of advertising the store should use. He has invited representatives from the local radio station, television station, and newspaper to make presentations in which they describe their audiences. The television station representative indicates that a TV commercial, which costs $15,000, would reach 25,000 potential customers. The breakdown of the audience is as follows:

| | Male | Female |
|---|---|---|
| Senior | 5,000 | 5,000 |
| Young | 5,000 | 10,000 |

The newspaper representative claims to be able to provide an audience of 10,000 potential customers at a cost of $4,000 per ad. The breakdown of the audience is as follows:

| | Male | Female |
|---|---|---|
| Senior | 4,000 | 3,000 |
| Young | 2,000 | 1,000 |

The radio station representative says that the audience for one of the station's commercials, which costs $6,000, is 15,000 customers. The breakdown of the audience is as follows:

| | Male | Female |
|---|---|---|
| Senior | 1,500 | 1,500 |
| Young | 4,500 | 7,500 |

The store has the following advertising policy:

- Use at least twice as many radio commercials as newspaper ads.
- Reach at least 100,000 customers.
- Reach at least twice as many young people as senior citizens.
- Make sure that at least 30% of the audience is female.

Available space limits the number of newspaper ads to seven. The store wants to know the optimal number of each type of advertising to purchase to minimize total cost.

Formulate a linear programming model for this problem.

20) Decision Variables are as follows:

| | Cost | Potential Customers |
|---|---|---|
| ($x_1$) TV Station ADs | 15 000 | 25 000 |
| ($x_2$) News-paper ADs | 4 000 | 10 000 |
| ($x_3$) Radio Station ADs | 6 000 | 15 000 |

Minimize $z = 15000x_1 + 4000x_2 + 6000x_3$

subject to $x_3 \geq 2x_2 \Rightarrow x_3 - 2x_2 \geq 0$

$25000x_1 + 10000x_2 + 15000x_3 \geq 100000$

$15000x_1 + 3000x_2 + 12000x_3 \geq 2.(10000x_1 + 7000x_2 + 3000x_3)$

$\Rightarrow -5000x_1 - 11000x_2 + 6000x_3 \geq 0$

$15000x_1 + 4000x_2 + 9000x_3 \geq 0{,}3.(25000x_1 + 10000x_2 + 15000x_3)$

$\Rightarrow 7500x_1 + 1000x_2 + 4500x_3 \geq 0$

$x_2 \leq 7$

$x_1, x_2, x_3 \geq 0$

9. The Shotz Beer Company has breweries in two cities; the breweries can supply the following numbers of barrels of draft beer to the company's distributors each month:

| Brewery | Monthly Supply (bbl) |
|---|---|
| A. Tampa | 3,500 |
| B. St. Louis | 5,000 |
| Total | 8,500 |

The distributors, which are spread throughout six states, have the following total monthly demand:

| Distributor | Monthly Demand (bbl) |
|---|---|
| 1. Tennessee | 1,600 |
| 2. Georgia | 1,800 |
| 3. North Carolina | 1,500 |
| 4. South Carolina | 950 |
| 5. Kentucky | 1,250 |
| 6. Virginia | 1,400 |
| Total | 8,500 |

The company must pay the following shipping costs per barrel:

| From | To (cost) 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| A | $0.50 | $0.35 | $0.60 | $0.45 | $0.80 | $0.75 |
| B | 0.25 | 0.65 | 0.40 | 0.55 | 0.20 | 0.65 |

Solve this problem by using the computer.

9) Decision Variables;

$x_{ij}$ : Number of barrels carried from Brewery $i$ to Distributor $j$

$i \in \{A, B\}$ ; $j \in \{1, 2, 3, 4, 5, 6\}$

Minimize

$$z = 0.50x_{A1} + 0.35x_{A2} + 0.60x_{A3} + 0.65x_{A4} + 0.80x_{A5} + 0.75x_{A6}$$
$$+ 0.25x_{B1} + 0.65x_{B2} + 0.40x_{B3} + 0.55x_{B4} + 0.20x_{B5} + 0.65x_{B6}$$

Subjet to

$$x_{A1} + x_{A2} + x_{A3} + x_{A4} + x_{A5} + x_{A6} = 3500$$
$$x_{B1} + x_{B2} + x_{B3} + x_{B4} + x_{B5} + x_{B6} = 5000$$
$$x_{A1} + x_{B1} = 1600$$
$$x_{A2} + x_{B2} = 1800$$
$$x_{A3} + x_{B3} = 1500$$
$$x_{A4} + x_{B4} = 950$$
$$x_{A5} + x_{B5} = 1250$$
$$x_{A6} + x_{B6} = 1400$$
$$x_{ij} \geq 0 \quad i = A, B \; ; \; j = 1, 2, 3, 4, 5, 6$$

23. Joe Henderson runs a small metal parts shop. The shop contains three machines—a drill press, a lathe, and a grinder. Joe has three operators, each certified to work on all three machines. However, each operator performs better on some machines than on others. The shop has contracted to do a big job that requires all three machines. The times required by the various operators to perform the required operations on each machine are summarized as follows:

| | (1) | (2) | (3) |
|---|---|---|---|
| Operator | Drill Press (min.) | Lathe (min.) | Grinder (min.) |
| 1 | 22 | 18 | 35 |
| 2 | 41 | 30 | 28 |
| 3 | 25 | 36 | 18 |

Joe Henderson wants to assign one operator to each machine so that the total operating time for all three operators is minimized.

Formulate a linear programming model for this problem.



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9) Decision Variables;

$$x_{ij} = \begin{cases} 1 & \text{if operator } i \text{ is assigned to job } j \\ 0 & \text{otherwise} \end{cases} \qquad \begin{matrix} i = 1,2,3 \\ j = 1,2,3 \end{matrix}$$

Minimize $z = 22x_{11} + 18x_{12} + 35x_{13} + 61x_{21} + 30x_{22} + 28x_{23} + 25x_{31} + 36x_{32} + 18x_{33}$

Subject to

$$\underbrace{\begin{aligned} x_{11} + x_{12} + x_{13} &= 1 \\ x_{21} + x_{22} + x_{23} &= 1 \\ x_{31} + x_{32} + x_{33} &= 1 \end{aligned}}_{\text{Each operator is assigned to one job}} \qquad \underbrace{\begin{aligned} x_{11} + x_{21} + x_{31} &= 1 \\ x_{12} + x_{22} + x_{23} &= 1 \\ x_{13} + x_{23} + x_{33} &= 1 \end{aligned}}_{\text{Each job is assigned to one operator}}$$

$x_{ij} \in \{0,1\}$ $i = 1,2,3$, $j = 1,2,3$

31. The manager of the Ewing and Barnes Department Store has four employees available to assign to three departments in the store—lamps, sporting goods, and linens. The manager wants each of these departments to have at least one employee, but not more than two. Therefore, two departments will be assigned one employee, and one department will be assigned two. Each employee has different areas of expertise, which are reflected in the daily sales each employee is expected to generate in each department, as follows:

| | Department Sales | | |
|---|---|---|---|
| Employee | 1. Lamps | 2. Sporting Goods | 3. Linens |
| 1 | $130 | $150 | $ 90 |
| 2 | 275 | 300 | 100 |
| 3 | 180 | 225 | 140 |
| 4 | 200 | 120 | 160 |

The manager wishes to know which employee(s) to assign to each department in order to maximize expected sales.

Formulate a linear programming model for this problem.

31) Maximize $z = 130x_{11} + 150x_{12} + \text{------} + 160x_{43}$

Subject to

$x_{11} + x_{12} + x_{13} = 1$

$x_{21} + x_{22} + x_{23} = 1$

$x_{31} + x_{32} + x_{33} = 1$

$x_{41} + x_{42} + x_{43} = 1$

$x_{11} + x_{21} + x_{31} + x_{41} \le 2$

$x_{12} + x_{22} + x_{32} + x_{42} \le 2$

$x_{13} + x_{23} + x_{33} + x_{43} \le 2$

$x_{ij} = 0$ or $1$



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